Curriculum Vitæ

Jérôme TAMBOUR

Current research themes: Non kähler complex manifolds, simplicial spheres, complex structure of topological spaces (moment-angle complexes,...), toric geometry.

PERSONAL PROFILE

Born on the 19^{th} of May 1984 (27 y.o.) in Auxerre (France).
French citizen.

CURRENT POSITION and CONTACT INFORMATIONS

Post-doctoral fellow at KAIST (Korean Advanced Institute for Science and Technology) in Daejeon (South Korea).
Under the supervision of Dan Zaffran.

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1. Articles and preprints

PhD thesis

[1] *Complexe manifolds and moment-angle complexes* (in french).
Available on http://tel.archives-ouvertes.fr.

Accepted papers

[2] *LVMB manifolds and simplicial spheres* (34 pages, in english), to appear in Annales de l'Institut Fourier. (preprint http://arxiv.org/abs/1006.1797)

Preprints

[3] *Linear Gale transform of a starshaped sphere* (21 pages, in english) (preprint http://arxiv.org/abs/1201.6205)

Miscellaneous

[4] *Ensembles de nombres* (197 pages, in french), a presentation of all the sets which deserves the name of "set of number" in mathematics. This includes integers, real and rational numbers, ordinals, cardinals, some algebras,... (available on my personal home page)

2. Research activities

Interests: Complex manifolds, algebraic topology, combinatorics (simplicial complexes), toric varieties, moment-angle complexes, torus manifolds.

Stay abroad

From 06/01/2012 to 06/02/2012	**Invitation in Osaka (Japan)** at the Osaka City University. *Hosts:* Mikiya Masuda.
From 04/15/2009 to 06/15/2009	**ECOS mission (cooperation program between France and South America)** at the U.N.A.M in Cuernavaca (Mexico). *Hosts:* Jose Seade and Alberto Verjovsky.
From 01/25/2009 to 02/18/2009	**Invitation in Vancouver (Canada)** at the PIMS (Pacific Institute for Mathematical Sciences). *Host:* Laurent Meersseman.

Participation to conferences, colloquia, workshops

From 01/12 to 05/12/2011	10^{th} **RIMS conference** Osaka City University (Japan).
From 28/11 to 30/11/2011	**Meeting "toric topology 2011"** Osaka City University (Japan).
From 18/07 to 20/07/2011	**Conference "Toric Methods in Homotopy Theory and Related Subjects".** Queen's University (Belfast, Royaume-Uni).
From 16/02 to 18/02/2011	**Mini-conference on "Hyperbolics complex spaces".** Institut de Mathématiques de Bourgogne. *4 lessons of 4 hours.*
September 2009	**Summer school "Uniformization of families of complex manifolds".** Institut de Mathématiques de Bourgogne. *6 lessons of 4 hours during 2 weeks.*
April 2009	**Inauguration of the ANR project "Complexe".** Institut de Mathématiques de Bourgogne.

(Past and future) communications

29/03/2012: **Seminar "Geometry and Dynamical systems"** at the Institut de Mathématiques de Bourgogne (Dijon, France).

26/03/2012: **Seminar "Analysis and Complex geometry"** at the Institut Elie Cartan (Nancy, France).

21/03/2012: **PhD students and post-doctoral fellows Seminar** at the Institut de Mathématiques de Bourgogne (Dijon, France).

01/2012: **Mini-course "Non kahler manifolds and toric geometry"** at Osaka City University (Japan).
4 lectures of 3 hours

30/11/2011 **Meeting "Toric geometry 2011"** at Osaka City University (Japan).

26/10/2011: **Seminar Topology and Geometry** at KAIST (South Korea).

20/07/2011: **"Toric Methods in Homotopy Theory and Related Subjects"** at the Queen's University in Belfast (United Kingdom).

28/06/2011: **Seminar Algebra and Number theory** at the Institute of Mathematics of Koeln (Germany).
Host: Stéphanie Cupit-Foutou.

31/05/2011: **Seminar Algebra, Geometry and Topology** at the Institut of Mathematics de Marseille (France).
Host: Martine Klughertz.

06/05/2010: **Day of the PhD Schools** of Dijon and Besançon (France).
Title: Geometry of moment-angle complexes.

15/04/2011: **Seminar Differential equations** at the Institut de Mathématiques de Toulouse (France).
Host: Martine Klughertz.

07/03/2011 **Seminar Algebra and Geometry** at the Institut Fourier (Grenoble, France).
Host: Mikhail Zaidenberg.

03/03/2011: **Seminar Groups, Algebra and Geometry** at the Laboratoire de Mathématiques de Poitiers (France).
Host: Frédéric Bosio.

25/02/2011: **Seminar Algebra and Geometry** at the Basel University (Switzerland).
Host: Pierre-Marie Poloni.

23/02/2011: **Seminar Dynamical Systems and Geometry** at the LAREMA (Laboratoire Angevin de Recherche en Mathématiques) of Angers (France).
Host: Jean-Jacques Loeb.

28/01/2011: **Seminar Algebraic geometry and Differential geometry** at the Laboratoire de Mathématiques de Brest (France).
Host: Guillaume Deschamps.

07/12/2010: **PhD Student seminar** at the Institut de Mathématiques de Bourgogne (France).
Title: Groups actions and construction of complex manifolds.

07/05/2010: **Day of the PhD Schools** of Dijon and Besançon (France).
Title: Non projective complex manifolds.

11/03/2010: **Seminar AGT (Algebra-Geometry-Topology)** at the Institut de Mathématiques de Bourgogne.
Title: Simplicial spheres and complex manifolds.

08/11/2009: **PhD Student seminar** at the Institut de Mathématiques de Bourgogne (France).
Title: Simplicial spheres and complex manifolds.

3. Summary of previous works

The main topic of my research is the study of the topology of a large family of nonkähler complex compact manifolds, known as LVMB manifolds. Let recall their origin and their construction. The intersections of quadrics with the following shape

$$\sum_{j=1}^{n} \lambda_j |z_j|^2 = 0, \qquad \sum_{j=1}^{n} |z_j|^2 = 1$$

where $\lambda_j \in \mathbb{R}^k$ have been studied since the 80's, especially by Camacho, Kuiper and Palis with the viewpoint of complex dynamical systems. In the case $k = 2$, the whole classification of the topology of these intersections (called *links*) has been made by Lopez de Medrano [LdM]. Later, Lopez de Medrano, Verjovsky (case $k = 2$) and Meersseman (general case, cf. [M]) showed that projectivizations of links can be endowed with a structure of complex compact manifolds and that these manifolds, called *LVM manifolds*, are either compact tori or nonkähler manifolds. They also showed that the classical examples of nonkähler manifolds, as Hopf manifolds and Calabi-Eckmann manifolds, can be constructed as particular cases of LVM manifolds. The LVMB manifolds are a generalization of LVM manifolds, with a strong combinatorial flavor. This is this generalization, due to Bosio [B], that we will introduce now.

We fixe some vectors $\lambda_1, \ldots, \lambda_n$ of the affine space $\mathbb{C}^m$, and we define an action of the complex Lie group $\mathbb{C}^* \times \mathbb{C}^m$ on $\mathbb{C}^n$ by

$$(*) \qquad (\alpha, T) \cdot z = (\alpha e^{<\lambda_1, T>} z_1, \ldots, \alpha e^{<\lambda_n, T>} z_n)$$

(where $<,>$ is the *non hermitian* usual inner product of $\mathbb{C}^m$, i.e. $< z, w >= \sum_{j=1}^{n} z_j w_j$).

This action is not free (since 0 is a fixed point). However, if we consider vectors $\lambda_1, \ldots, \lambda_n$ with additional (but very general) conditions and if we restrict the action to some dense open subset $\mathcal{S}$ of $\mathbb{C}^n$, the action is free and proper and the orbit space $\mathcal{N}$ is compact and can be endowed with a structure of complex manifold. Such a manifold is called a *LVMB manifold.* Moreover, as for the LVM manifolds, we have the following alternative for LVMB manifolds: they are either nonkähler manifolds or compact tori. In the same article [B], Bosio prove that if $m = 1$, then the manifolds we obtain are analytic deformations of LVM manifolds. Yet, two natural questions were left open in Bosio's article:

Question 1: Does there exist a LVMB manifold whose complex structure is different from the complex structures of LVM manifolds ?

Question 2: Does there exist a LVMB manifold whose topology is different from the topologies of LVM manifolds ?

In [CFZ], Cupit-Foutou and Zaffran give an affirmative answer to the first question. Their method consists in showing that in a generic case, LVMB manifolds are a fibration on toric complete varieties. For they show that if the vectors $\lambda_1, \ldots, \lambda_n$ verify the condition (K), that is their coordinates are rationals (or more generally their image by a real affine automorphism of $\mathbb{C}^m$ have integer coordinates), then the action $(*)$ of $\mathbb{C}^* \times \mathbb{C}^m$ on $\mathcal{S}$ described as above can be seen as the restriction of an algebraic action of $(\mathbb{C}^*)^{2m+1}$ on $\mathcal{S}$. Using arguments of Mumford's Geometric Invariant Theory, they show that the orbit space for this action is a toric complete variety. In the case od a LVM manifold, the toric variety they obtained is projective whereas it is not necessarily the case for general LVMB manifolds.

The aim of my research is to study in details the topology of LVMB manifolds. The main lead I am following is to use the combinatorial properties of the LVMB manifolds and toric varieties. Indeed, many properties of a LVMB manifold $\mathcal{N}$, such that several of its topological invariants (such as cohomology rings with real or integer coefficients), can be determined by the the properties of a simplicial complex $\mathcal{P}$, naturally associated to $\mathcal{N}$. This *associated complex* is the generalization of the associated polytope of a LVM manifold (cf. [M] or [BM]).

I showed in [2] (cf. section 1. Articles and preprints) that this associated complex is a rationally starshaped simplicial sphere (that is the underlying complex of a simplicial complete fan). I also proved in the same article that every such sphere appears as the associated complex of some LVMB manifold. On the other hand, I stated that the orbit space of $\mathcal{S}$ for the restriction of the action $(*)$ restricted to $\mathbb{R}_+^* \times \mathbb{C}^m$

is a moment-angle complex, a topological object studied in particular by Davis, Januszkiewicz, Buchstaber and Panov (cf. [BP] for a complete description of those complexes). Moment-angle complexes have a very strong combinatorial nature (they are parametrized by simplicial complexes) and play in topology the same part of very large family of examples as the toric varieties in algebraic geometry. Until recently, only few was known about the existence of smooth structure or complex structure on moment-angle complexes. The study in the article [2] implies that moment-angle complexes parametrized by rationally starshaped simplicial spheres can be endowed with a structure of LVMB manifolds.

At last, I used the preceding results to study the topology of several LVMB manifolds. In particular, I studied LVMB manifolds associated to non polytopal simplicial spheres. My choice was to study the simplest examples (in terms of dimension and number of vertices), that are the Brückner sphere and the Barnette sphere (they are 3-dimensional spheres with 8 vertices). For, I have constructed a rationally starshaped realization of these spheres and I have designed algorithms (with the software Maple) with the view to compute their homology and cohomology. The results are presented in my thesis [1]. In particular, the LVMB manifolds associated to these spheres have the same cohomology rings as some LVM manifolds.

4. Research plan

In my thesis, I have deepened the study of the LVMB manifolds and strengthened their relations to important objects in other domains: toric varieties, moment-angle complexes, triangulation of spheres,.... There are several ways to continue this investigation.

Topology of the LVMB manifolds

The first one would consist in response to the question 2 in the previous section. If the answer is affirmative, a concrete example will be obtained from a rationally starshaped sphere which is not polytopal. The simplest examples of such a sphere are the Brückner sphere and the Barnette sphere but, even for these simple examples, the computation leaded in my thesis show that the comparison of the topologies is not an easy task. Theoretically, there are many other examples of non polytopal spheres. However, the description of simplicial spheres is complete only for small dimension or small number of vertices (for instance, combinatorial 3-manifolds with 10 vertices have been classified only in 2008).

In a similar direction, the second idea would be to find an answer to the following question: is the smooth structure obtained on moment-angle complexes parametrized by rationally starshaped spheres unique? If we restrict our attention to the polytopal case and to smooth structures which respect the torus action, then the answer is affirmative (cf. [BM]). This research could be the study of other topological invariant of moment-angle complexes. In particular, it would be very interesting to express the Steenrod squares of a moment-angle complexe in terms of the combinatoric of the simplicial complex which describe it. This could lead to an answer to the question 2. Indeed, studying the moment-angle complexes associated to the Brückner and Barnette spheres can be reduced to the study of 5-connected smooth manifolds of dimension 12. Yet, these manifolds have been classified by Wall and Ishimoto, and the Steenrod squares influence considerably in the classification of Ishimoto.

Another approach would be to use several combinatorial tools to study properties of LVMB manifolds. A promising approach in this path would be to seek how to characterize topologically LVMB manifold varieties derived from a PL sphere, i.e. a simplicial complex admitting a subdivision combinatorially equivalent to some subdivision of the boundary of a simplex (cf. [BP]). Such spheres are obtained from simple operations called bistellar moves. At the moment-angle complex level, these moves correspond to bistellar surgeries. Better understanding of these moves would lead to the construction of new explicit examples of LVMB manifolds. The effect of bistellar moves on moment-angle complexes has already been partially studied in [BP] and in [LdMG].

Geometric structures of moment-angle complexes

Moreover, my construction give an idea to answer to the following question:

Question 3: Describe the class of simplicial complexes such that the associated moment-angle complex can be endowed with a smooth structure. Same question for the complex structure.

This question is of great importance in toric topology, evidenced by the prolific literature on the subject in recent years. Mentions may be made of [PU], [?], [?] or [?]. The obvious idea is to study the topology of moment-angles complexes associated to non starshaped spheres. Many non starshaped spheres can be obtained by taking connected sum of two starshaped spheres (see [ES] for such kind of examples). This type of non starshaped spheres seems to be a good playground to try to enlarge the family of moment-angle complexes which admit a complex structure; or to find obstruction to this construction. Very recently, Matsumura and Moore showed in [MM] that a necessary condition a simplicial complex must satisfies for its associated moment-angle complex to have a smooth (namely, being Gorenstein*) is also satisfied by the connected sum of two Gorenstein* complexes. At KAIST, I started a joint work with Matsumura on how to generalize its notion of connected sum to simplicial posets and how this generalization is related to the geometry of torus manifolds (a big family of smooth manifolds with an effective action of a torus and nonempty set of fixed point). The family of torus manifolds contains the family of smooth toric varietes and seems to be a natural candidate for being the base space of a generalization of the Calabi-Eckmann fibration, the total space being a moment-angle complex associated to a simplicial poset.

Application to combinatorics

Finally, in the different chapters of my thesis, as well as numerous articles on the LVMB manifolds, the combinatorics of simplicial complexes was used to establish properties of LVMB manifolds. A last idea of development would be making the opposite approach: using the properties of LVMB manifolds to derive combinatorial results about simplicial complexes or simplicial triangulations of spheres.

For example, in the rational case (i.e. when the parameters describing the LVMB manifolds are integer), a LVMB manifold is the total space of a Calabi-Eckmann fibration over a compact toric variety. It is well-known that the even Betti numbers of such a variety correspond exactly to the h-vector of the underlying complex of the variety. In [BZ], Battaglia and Zaffran studied the non rational case. In this case, a LVMB manifold admits a foliation whose closures of leaves are compact torus (in the rational case, all leaves are closed and the leaf space can be identified with the toric variety). They show in [BZ] that, for LVMB manifold corresponding to a shellable starshaped sphere, the odd Betti numbers for the basic cohomology of the foliation vanish and the even Betti number coincide with the h-vector of the sphere. As a corollary, they obtain a new proof of Dehn-Sommerville equations and of the positivity of the g-vector for shellable starshaped spheres.

They use in their computation a covering of the LVMB manifold by suitably chosen foliated open subsets and the compute the baic cohomology of the foliation using a Mayer-Vietoris sequence and the order given by a shelling of the sphere. In a recent project with Zaffran, I try to generalize this computation to any starshaped sphere using spectral sequences for foliations.

References

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[BP] V.M. Buchstaber and T.E. Panov. *Torus actions and their applications in topology and combinatorics*, volume 24 of *University Lecture Series*. American Mathematical Society, Providence, RI, 2002.

[BZ] F. Battaglia and D. Zaffran. Foliations modeling nonrational simplicial toric varieties. arXiv:1108.1637v1.

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[LdM] S. López de Medrano. Topology of the intersection of quadrics in $\mathbf{R}^n$. In *Algebraic topology (Arcata, CA, 1986)*, volume 1370 of *Lecture Notes in Math.*, pages 280–292. Springer, Berlin, 1989.

[LdMG] S. Lopez de Medrano and S. Gitler. Intersections of quadrics, moment-angle manifolds and connected sums. arXiv:0901.2580v2.

[M] L. Meersseman. A new geometric construction of compact complex manifolds in any dimension. *Math. Ann.*, 317(1):79–115, 2000.

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[PU] T. Panov and Y. Ustinovsky. Complex-analytic structures on moment-angle manifolds. arXiv:1008.4764v1.